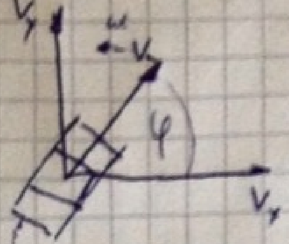
Zustandsübergang

$$X_n = \begin{bmatrix} x_n \\ y_n \\ v_n \\ \varphi_n \\ \omega_n \end{bmatrix} \quad \begin{bmatrix} ? \\ ? \\ v_n \\ \varphi_n + \omega_n T \\ \omega_n \end{bmatrix} = X^-_{n+1}$$



"constant turn rate and velocity"

CTRV

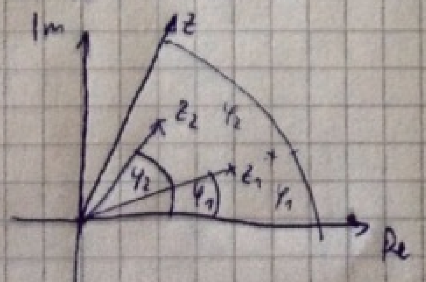
~~Drehung in~~

Grundsätzlich:
$$s_{n+1} = s_n + \int_{t_n}^{t_{n+1}} v_n(t)\,dt$$

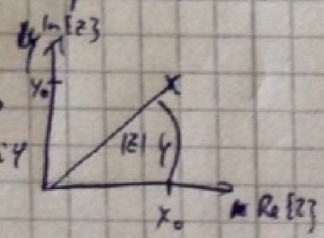
2-Dimensional ...

Ebene Drehungen in der Ebene? $\longrightarrow$ Komplexe Zahlen

$$z = z_1 \cdot z_2 = |z_1||z_2| \cdot e^{i(\varphi_1+\varphi_2)}$$



$$z_0 = x_0 + i y_0 = \sqrt{x_0^2 + y_0^2} \cdot e^{i\varphi}$$



$$\underline{v}_n^{(t)} = v_{x,n} + i\,v_{y,n} = v_n \cdot e^{i(\varphi+\omega t)}$$

$$\underline{s}_n = x_n + i\,y_n$$

$$\underline{s}_{n+1} = \underline{s}_n + \int_{t_n}^{t_{n+1}} v_n \cdot e^{i(\varphi+\omega t)}\,dt$$

CTRV: konstant (konstant) ω konstant

$$\underline{S}_{n+1} = \underline{S}_n + v_n \cdot e^{i\varphi} \underbrace{\int_{t_n}^{t_{n+1}} e^{i\omega t}\, dt}$$

$$\left[\frac{1}{i\omega} e^{i\omega t} \right]_{t_n=0}^{t_{n+1}=T} = \frac{+i}{\omega} \left(-e^{i\omega T} + 1 \right)$$

$$= \underline{S}_n + \frac{v_n}{\omega_n} \cdot i \left(e^{i\varphi_n} - e^{i(\omega_n T + \varphi_n)} \right) = x_{n+1} + i y_{n+1}$$

$$= x_n + i y_n + \frac{v_n}{\omega_n} \left(i \cos\varphi_n - \sin\varphi_n - i\cos(\omega_n T + \varphi_n) + \sin(\omega_n T + \varphi_n) \right)$$

$$x_{n+1} = x_n + \frac{v_n}{\omega_n} \left(-\sin\varphi_n + \sin(\omega_n T + \varphi_n) \right)$$

$$y_{n+1} = y_n + \frac{v_n}{\omega_n} \left(\cos\varphi_n - \cos(\omega_n T + \varphi_n) \right)$$

$$X_{n+1} = \begin{bmatrix} x_n + \frac{v_n}{\omega_n} \left(\sin(\omega_n T + \varphi_n) - \sin\varphi_n \right) \\ y_n + \frac{v_n}{\omega_n} \left(\cos\varphi_n - \cos(\omega_n T + \varphi_n) \right) \\ v_n \\ \varphi_n + \omega_n T \\ \omega_n \end{bmatrix}$$

Alternativ

$$\underline{X}_n = \begin{bmatrix} \underline{S}_n \\ \underline{V}_n \\ \omega_n \end{bmatrix} = \begin{bmatrix} x_n + i y_n \\ v_n \cdot e^{i\varphi_n} \\ \omega_n \end{bmatrix}$$

$$\underline{X}_{n+1} = \begin{bmatrix} \underline{S}_n + i \frac{v_n}{\omega_n} \cdot \left(1 - e^{i\omega_n T} \right) \\ \underline{V}_n \cdot e^{i\omega_n T} \\ \omega_n \end{bmatrix}$$